

Calculation of three-body nuclear reactions with angular-momentum and parity-dependent optical potentials

A. Deltuva* and D. Jurčiukonis

Institute of Theoretical Physics and Astronomy, Vilnius University, Saulėtekio al. 3, LT-10222 Vilnius, Lithuania

Angular-momentum or parity-dependent nonlocal optical potentials for nucleon- ^{16}O scattering able to fit differential cross section data over the whole angular regime are developed and applied to the description of deuteron- ^{16}O scattering in the framework of three-body Faddeev-type equations for transition operators. Differential cross sections and deuteron analyzing powers for elastic scattering and $^{16}\text{O}(d,p)^{17}\text{O}$ transfer reactions are calculated using a number of local and nonlocal optical potentials and compared with experimental data. Angular-momentum or parity-dependence of the optical potential turns out to be quite irrelevant in the considered three-body reactions while nonlocality is essential for a successful description of the differential cross section data, especially in transfer reactions.

PACS numbers: 24.10.-i, 21.45.-v, 25.45.Hi, 25.40.Hs

I. INTRODUCTION

Deuteron scattering from composite nuclei is, in general, a many-body problem, but very often it is treated as three-body problem in a system consisting of proton (p), neutron (n), and inert nuclear core A [1–3]. The interactions between nucleons and core are modeled by effective optical potentials (OPs); there is a large variety of phenomenological parametrizations [4–8]. In some cases such models are successful in describing the experimental data, and in some cases fail, calling for improvements in the OP or the dynamical model, e.g., by including the excitation of the core [9]. In particular cases the failures can be seen already in the nucleon-nucleus two-body system. For example, standard optical potentials were found to be unable to account for precise large-angle nucleon-nucleus elastic differential cross section data above 20 MeV lab energy for stable closed-shell nuclei such as ^{16}O or ^{40}Ca [10]. This is not very surprising given significant momentum transfer in that regime and possible excitations. It was argued [11, 12] that these effects give rise to parity- (π) or angular-momentum (L) dependent components in the OP. Indeed, with such additional terms the description of large-angle differential cross section data was significantly improved [10, 13]. Naturally one may raise the question what consequences the improvements and new OP terms have on three-body reaction observables. Unfortunately, explicitly π - or L -dependent OPs are not suitable for standard practical calculations within distorted wave Born approximation (DWBA), adiabatic wave approximation (ADWA) and continuum discretized coupled channels (CDCC) frameworks. However, exact Faddeev-type theory, implemented in individual partial-wave representations for all three involved pairs, is capable of using such π - and L -dependent potentials. Thus, the aim of the present work is to study three-body nuclear reactions with π - and L -dependent

OPs. We choose $d + ^{16}\text{O}$ elastic and transfer reactions as a working example. However, all existing π - and L -dependent OP parametrizations for nucleon- ^{16}O are local [10, 13], while in Ref. [14] it was found that the Perey-Buck-type exchange nonlocality of the OP is important in three-body reactions, especially for transfer. For this reason we create several nonlocal parametrizations of π - and L -dependent OPs, fitted to the same experimental data [15, 16] as the local ones [10, 13].

In Sec. II we describe the employed two-body nucleon- ^{16}O potentials, and in Sec. III the three-body scattering equations. Results are presented in Sec. IV, while summary is given in Sec. V.

II. NUCLEON-NUCLEUS POTENTIALS

A. Angular-momentum-dependent optical potential

We start with a nonlocal OP of the form proposed by Giannini and Ricco [17] and augment it with an L -dependent part, resulting

$$\begin{aligned} V_L(\mathbf{r}', \mathbf{r}) = & -H_c(x)[V_V f_V(y) + iW_V f_W(y) + iW_S g_S(y)] \\ & -H_s(x)V_s \frac{2}{y} \frac{df_s(y)}{dy} \boldsymbol{\sigma} \cdot \mathbf{L} \\ & -H_c(x)[\tilde{V} g_{\tilde{V}}(y) + i\tilde{W} g_{\tilde{W}}(y)] f_L(L^2). \end{aligned} \quad (1)$$

Here V_i and W_i are potential strengths for various real and imaginary terms, while for each term the shape is given by

$$x = |\mathbf{r}' - \mathbf{r}|, \quad (2a)$$

$$y = |\mathbf{r}' + \mathbf{r}|/2, \quad (2b)$$

$$H_i(x) = (\pi\beta_i^2)^{-3/2} e^{-x^2/\beta_i^2}, \quad (2c)$$

$$f_i(y) = [1 + e^{(y-R_i)/a_i}]^{-1}, \quad (2d)$$

$$g_i(y) = -4a_i df_i(y)/dy = 4f_i(y)[1 - f_i(y)] \quad (2e)$$

* arnoldas.deltuva@tfai.vu.lt

with radius R_i , diffuseness a_i , and nonlocality parameter β_i . The dependence on the orbital angular momentum L in the form of Woods-Saxon function $f_L(L^2)$ given in Eq. (2d) is taken over from Ref. [10] where local L -dependent potential was constructed. We determine potential strength parameters by fitting theoretical $p + {}^{16}\text{O}$ predictions to experimental data for the differential cross section $d\sigma/d\Omega$, total inelastic cross section, and proton analyzing power A_y from Refs. [15, 16], i.e., same data that constrain the local OP of Ref. [10]. In addition, also geometric parameters R_i , a_i , and β_i have been varied, typically within 10% of original values [17], to improve the fit quality. The parameters defining the L -dependence turn out to be comparable to the local case [10], i.e., typically $3 < \sqrt{R_L} < 4$ and $0.5 < \sqrt{a_L} < 0.7$. Note that some local OP parameters in Ref. [10] depend strongly on the collision energy and that dependence is not smooth, reflecting the fact that backward-angle experimental data exhibit non-monotonic energy dependence, probably due to the presence of resonant proton-nucleus states. We emphasize that in three-body reactions the energy of each interacting two-body subsystem formally runs from the available three-body energy to $-\infty$, but, in order to have a single three-body Hamiltonian and thereby preserve a Hamiltonian theory [18], it is preferable to use two-body potentials with fixed sets of parameters; the results in Sec. III are obtained following this strategy. The ability of the OP to account for the two-body reaction data over a broader energy regime may be important for its success in three-body reactions and deserves investigation. The nonlocal OP, at least to some extent, is able to absorb smooth energy dependence of data into nonlocality, but far less a non-smooth behavior. Thus, we have not achieved a single parameter set describing the data over the full angular regime at all energies. In fact, the analyzing power data are only accounted for center-of-mass (c.m.) scattering angles $\Theta_{\text{c.m.}}$ up to about 100° . Nevertheless, when we fit the experimental data at a given energy, the resulting OP reproduces well the data for other energies at $\Theta_{\text{c.m.}} \leq 90^\circ$ and only fails at backward angles where the differential cross section is very small. This is an important improvement as compared to the local OP of Ref. [10] that yields considerably worse description for the data not included in the fit. An example is shown in Figs. 1 and 2 where the predictions using local and nonlocal L -dependent OPs, determined solely by the data at proton lab energy $E_p = 27.3$ MeV, are compared with data at $E_p = 27.3$ and 34.1 MeV. Thus, Fig. 1 reflects the quality of the fit at a single given energy, while Fig. 2 reflects the predictive power of fixed-energy OPs for energies not included in the fit. In this latter case nonlocal OPs are more successful, indicating weaker energy dependence of their parameters as compared to the local OP. We present also predictions of parity-dependent OP from the next subsection, V_π , and of nonlocal L -independent OP with parameters from Ref. [14], labeled V_N . The latter was not properly fitted to the present data failing

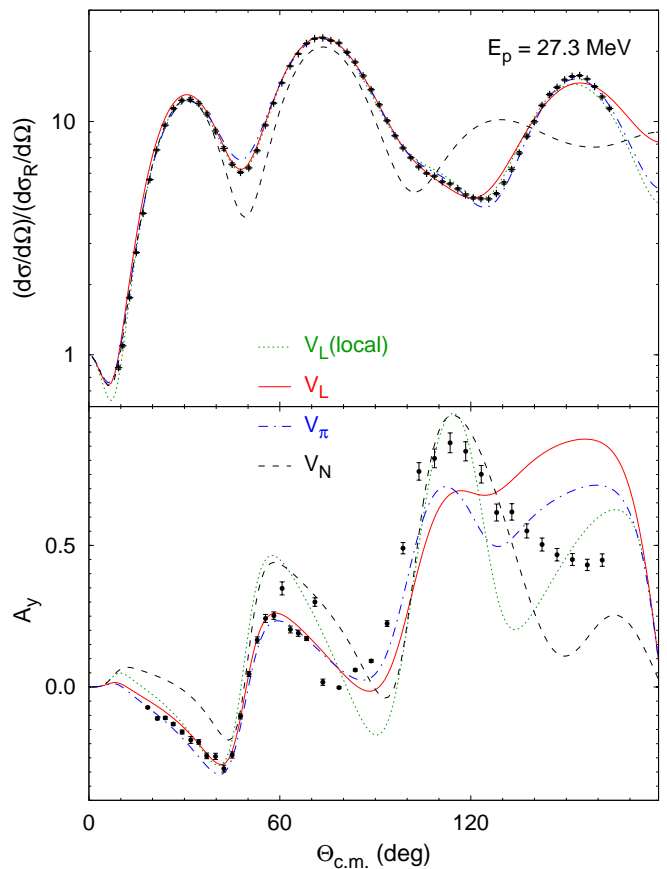


FIG. 1. (Color online) Differential cross section divided by Rutherford cross section and proton analyzing power for $p + {}^{16}\text{O}$ elastic scattering at $E_p = 27.3$ MeV. Parameters of local L -dependent OP from Ref. [10] (dotted curves), nonlocal L -dependent OP (solid curves) and nonlocal π -dependent OP (dashed-dotted curves) are fitted to experimental $E_p = 27.3$ MeV data. Results for nonlocal L -independent OP (dashed curves) without proper fit are shown as well. The data are from Refs. [15, 16].

at backward angles and, for A_y , also at forward angles $\Theta_{\text{c.m.}} \leq 40^\circ$, but otherwise provides a rough description of the experimental data. To confirm the conclusion on the superiority of nonlocal L -dependent OP in the two-body system, we created a number of parametrizations [19].

As for $n + {}^{16}\text{O}$ scattering, the available experimental data [20–22] are scarcer and less precise. We tried two options for L -dependent $n + {}^{16}\text{O}$ potential: i) taking over the parameters of the $p + {}^{16}\text{O}$ potential, ii) explicitly fitting to $n + {}^{16}\text{O}$ experimental data. An example for $n + {}^{16}\text{O}$ scattering at $E_n = 24$ MeV neutron energy is presented in Fig. 3. While for the differential cross section the quality is nearly the same in both cases, explicit fitting leads to a better description of the neutron analyzing power. We show results for two sets of parameters to demonstrate large model dependence for backward angle

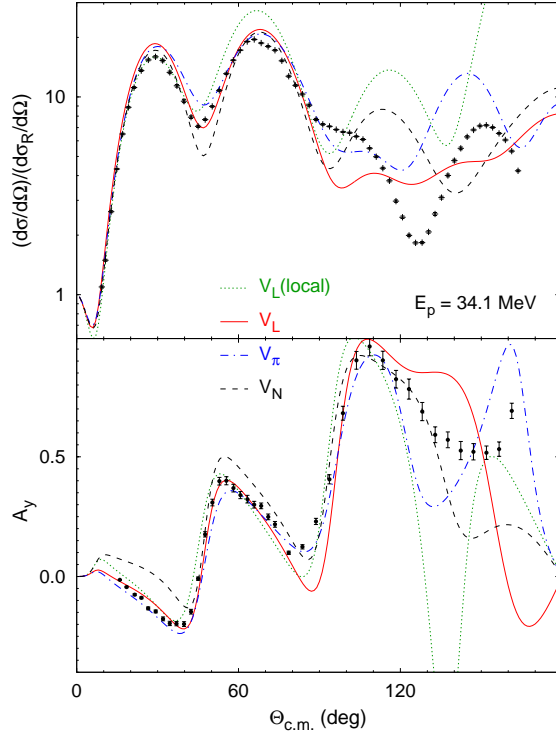


FIG. 2. (Color online) Differential cross section divided by Rutherford cross section and proton analyzing power for $p + {}^{16}\text{O}$ elastic scattering at $E_p = 34.1$ MeV. Curves are as in Fig. 1 with OP parameters fitted to experimental $E_p = 27.3$ MeV data. The results are not constrained in any way by the shown $E_p = 34.1$ MeV data from Refs. [15, 16].

A_y .

B. Parity-dependent optical potential

The mechanism giving rise to parity-dependent terms in the OP [12, 13, 23] is different from that of L -dependent terms, but nevertheless has led to a comparable quality when fitting the data. To get the nonlocal version we replace the $f_L(L^2)$ term in Eq. (1) by a parity-dependent term, resulting in a nonlocal π -dependent OP

$$\begin{aligned} V_\pi(\mathbf{r}', \mathbf{r}) = & -H_c(x)[V_V f_V(y) + iW_V f_W(y) + iW_S g_S(y)] \\ & -H_s(x)\frac{2}{y}\left[V_s\frac{df_s(y)}{dy} + iW_s\frac{df_{s_i}(y)}{dy}\right]\boldsymbol{\sigma} \cdot \mathbf{L} \\ & -(-1)^L H_c(x)[\tilde{V}_V f_{\tilde{V}}(y) + i\tilde{W}_V f_{\tilde{W}}(y) \\ & + \tilde{V}_S g_{\tilde{S}_r}(y) + i\tilde{W}_S g_{\tilde{S}_i}(y)]. \end{aligned} \quad (3)$$

In addition, we allowed for an imaginary spin-orbit term with strength W_s , following Ref. [13] where π -dependent local OP was developed, but fixed nonlocality parameters β_i to original values from Ref. [17]. Otherwise the fitting procedure is the same as in previous subsection,

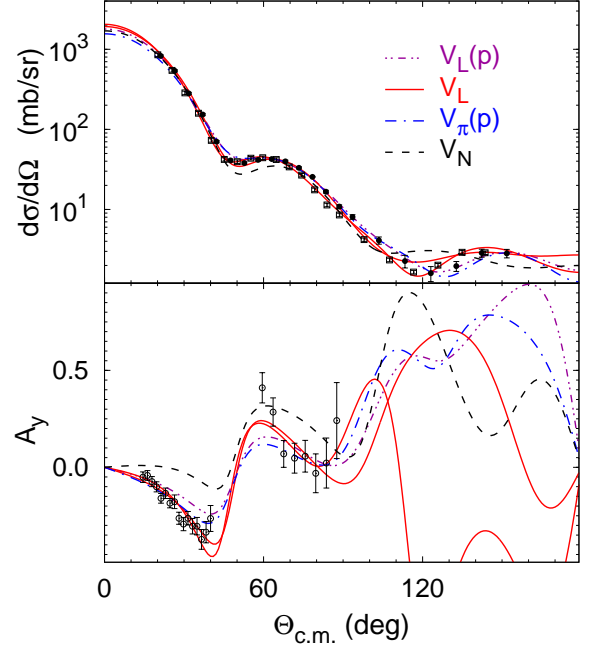


FIG. 3. (Color online) Differential cross section and neutron analyzing power for $n + {}^{16}\text{O}$ elastic scattering at $E_n = 24$ MeV. Results obtained with various nonlocal potentials are compared with the experimental data from Refs. [20] (\bullet), [22] (\square), and [21] (\circ). The predictions with $p + {}^{16}\text{O}$ potential parameters from Fig. 1 are denoted by (p).

and the achieved quality in describing experimental data is comparable to that of L -dependent OP as can be seen in Fig. 1. Worth noting are different radial shapes for π -dependent and independent terms, in particular significantly smaller diffuseness $a_{\tilde{V}}$ as compared to a_V , consistently with findings of Ref. [12].

C. Discussion

L - and π -dependence of the OP is not really surprising, since OP describes not a fundamental interaction but an effective one between the nucleon and composite nucleus. Internal degrees of freedom of the nucleus A , if taken into account explicitly, would lead to a highly complicated effective two-body nucleon-nucleus interaction. Solving the $(A+1)$ -nucleon scattering problem with sufficient accuracy is beyond the present capabilities, but attempts have been made to justify the nonstandard OP terms by the effect of simplest internal degrees of freedom of the nuclear core. E.g., Ref. [12] argues that contributions of the core excitation can be approximated by π -dependent terms, while Refs. [10, 11] relate the L -dependence of the OP to the deuteron channel coupling. For curiosity we verified this concept in a toy model using theoretical results from Ref. [14] for proton elastic scattering on ${}^{16}\text{O}$ and ${}^{17}\text{O}$, but we expect it to be qualitatively valid for any nuclei

$(A-1) \equiv B$ and A . Starting with an L -independent OP for $p+B$, a real binding potential for $n+B$, and a realistic $n+p$ potential having central, spin-spin, spin-orbit, and tensor terms, results for $p+A$ were obtained by solving exact three-body equations, thereby including $p+n+B$ breakup and $d+B$ transfer channels to all orders. The resulting three-body $p+A$ elastic cross section could not be fitted well with the two-body standard OP for $p+A$, but the inclusion of L -dependent terms in the OP for $p+A$, i.e., $p+^{17}\text{O}$ in case of Ref. [14], significantly improved the fit. Obviously, such an approach is not reliable for a quantitative determination of the OP as it takes into account only one-neutron internal degrees of freedom in the core and relies on the potentials for the $p+B$ and $n+B$ subsystems (that may be L -dependent themselves), but it demonstrates that the L -dependence of the OP appears. Of course, L -independent phenomenological OPs do not exclude the coupling to the deuteron channel, but include it implicitly in an L -averaged way by fitting the data. Applying an L -dependent potential in a three-body $p+n+A$ system with present deuteron channel is justified in exact calculations dealing with three pairwise $p+A$, $n+A$, and $n+p$ interactions, but may lead to double counting in DWBA-type approaches that generate the $p+(A+n)$ wave not through a rigorous solution of the three-body problem but from a $p+(A+n)$ two-body OP. We also expect that if one would attempt to calculate deuteron-nucleus two-body OP starting from a three-body problem with L -independent nucleon-nucleus OPs, the resulting OP in a similar way should acquire L -dependence.

III. THREE-BODY SCATTERING EQUATIONS

We describe deuteron-nucleus reactions in the framework of exact Faddeev-type three-body scattering equations. We use Alt-Grassberger-Sandhas (AGS) integral equations [24] for transition operators

$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\gamma} \bar{\delta}_{\beta\gamma} T_{\gamma} G_0 U_{\gamma\alpha}, \quad (4)$$

with $\bar{\delta}_{\beta\alpha} = 1 - \delta_{\beta\alpha}$, free resolvent $G_0(Z) = (E + i0 - H_0)^{-1}$, three-particle relative motion kinetic energy operator H_0 and available energy E , and two-body transition matrix

$$T_{\gamma} = v_{\gamma} + v_{\gamma} G_0 T_{\gamma}. \quad (5)$$

The latter is calculated for each pair γ with the corresponding two-body potential v_{γ} , where in the odd-man-out notation v_1 denotes the interaction within the pair (23) and so on. On-shell matrix elements of $U_{\beta\alpha}$ taken between the corresponding channel states are reaction amplitudes needed for the calculation of scattering observables.

We solve AGS equations in the momentum-space partial-wave representation. We employ three complete sets of base functions

$|p_{\alpha} q_{\alpha} (l_{\alpha} \{ [L_{\alpha} (s_{\beta} s_{\gamma}) S_{\alpha}] j_{\alpha} s_{\alpha} \} S_{\alpha}) J M \rangle$. Here $(\alpha\beta\gamma) = (123)$, (231) , or (312) , p_{α} is magnitude of relative momentum within pair $(\beta\gamma)$, q_{α} is magnitude of relative momentum between spectator α and c.m. of pair $(\beta\gamma)$, L_{α} and l_{α} are orbital angular momenta associated with p_{α} and q_{α} , respectively, and $s_{\alpha}, s_{\beta}, s_{\gamma}$ are spins of the corresponding particles. All discrete angular momentum quantum numbers are coupled to total angular momentum J with the projection M , while S_{α} , j_{α} and S_{α} are angular momenta of intermediate subsystems. Using all three sets $\alpha = 1, 2$, and 3 of these basis states allows the calculation of each potential v_{α} and transition matrix T_{α} in its proper basis. Obviously, this enables easy inclusion of L - and π -dependent potentials, in contrast to CDCC and other approximations, where only one set of base functions is being used.

The proton-nucleus Coulomb force is included via the screening and renormalization method [25–27]. For $d+^{16}\text{O}$ elastic and transfer reactions we obtain well-converged results with Coulomb screening radius around 10 or 12 fm, and including $J \leq 30$ states with L_{α} up to 3, 8, and 14 for $n+p$, $n+^{16}\text{O}$, and $p+^{16}\text{O}$ pairs, respectively. For the $n+p$ interaction we take the realistic CD Bonn potential [28] and use potentials from previous section for nucleon-nucleus pairs.

IV. RESULTS

Using nucleon-nucleus OPs from Sec. II and Refs. [10, 17] and the realistic neutron-proton CD Bonn potential [28] we study $d+^{16}\text{O}$ elastic scattering and transfer reactions $^{16}\text{O}(d,p)^{17}\text{O}$. In the former case there exist differential cross section and deuteron analyzing power data at $E_d = 56$ MeV deuteron lab energy [29]. Comparison of those experimental data and our predictions, including four L -dependent and three π -dependent nonlocal models, is presented in Fig. 4. One can notice immediately that the local L -dependent OP from Ref. [10], although being successful in $p+^{16}\text{O}$ scattering, fails heavily at large angles in $d+^{16}\text{O}$ scattering, strongly overpredicting the differential cross section. In contrast, nonlocal models, both L - or π -dependent, slightly underpredict the differential cross section at large angles, but quite reasonably follow its shape. Properly fitted L - and π -dependent models, both local and nonlocal, provide a reasonable description of deuteron analyzing powers up to $\Theta_{\text{c.m.}} = 40^\circ$ or 60° (in some cases, with exception of $\Theta_{\text{c.m.}} = 20^\circ$ where $d\sigma/d\Omega$ has a deep minimum), but deviate from data and from each other at larger scattering angles. The L -independent nonlocal OP [17] accounts for cross section data with a quality comparable to nonlocal L - and π -dependent OPs, but fails for the deuteron vector analyzing power A_y at $20^\circ \leq \Theta_{\text{c.m.}} \leq 40^\circ$. This is expected given that it was not fitted to $p+^{16}\text{O}$ A_y data and shows there a similar discrepancy. Quite surprisingly, the description of all measured deuteron tensor analyzing powers A_{yy} , A_{xx} , and A_{xz} using this model [17] turns out to

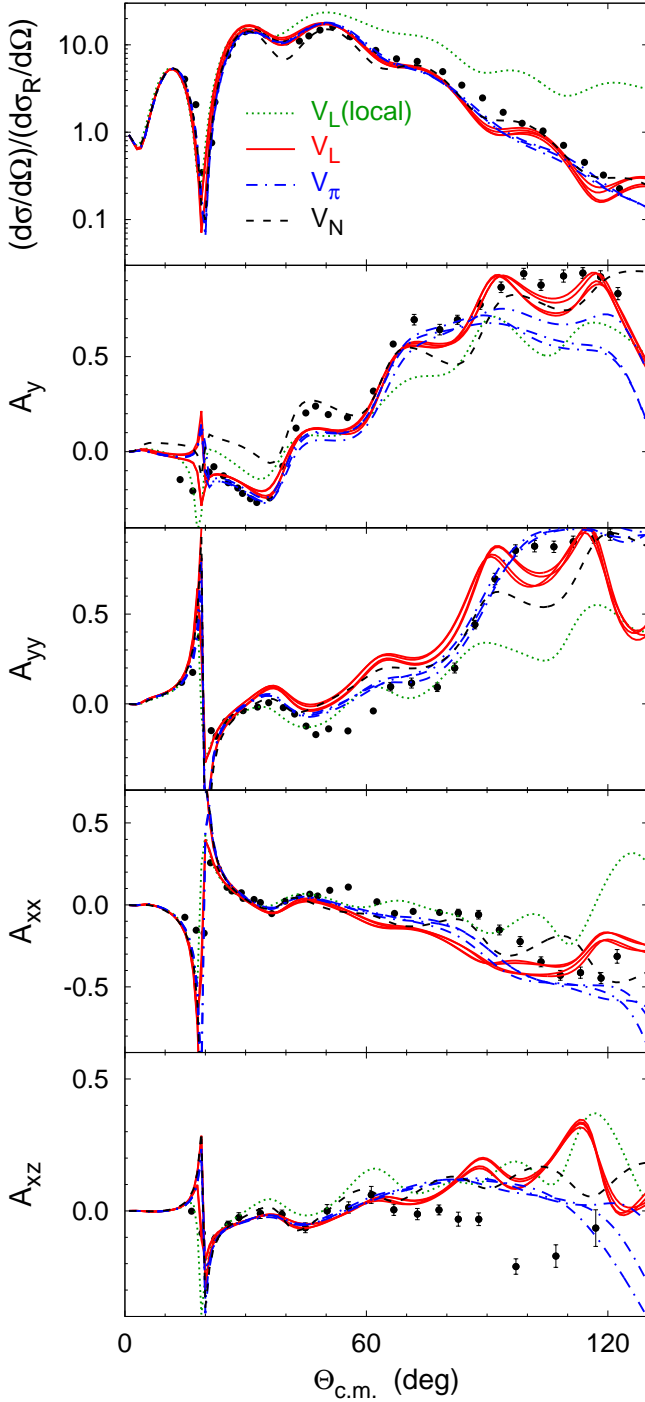


FIG. 4. (Color online) Differential cross section divided by Rutherford cross section and deuteron analyzing powers for $d + {}^{16}\text{O}$ elastic scattering at $E_d = 56$ MeV. Predictions obtained using local L -dependent OP from Ref. [10] (dotted curves), four parametrizations of nonlocal L -dependent OP (four solid curves), three parametrizations of nonlocal π -dependent OP (three dashed-dotted curves), and nonlocal L -independent OP (dashed curves) are compared with the experimental data from Ref. [29].

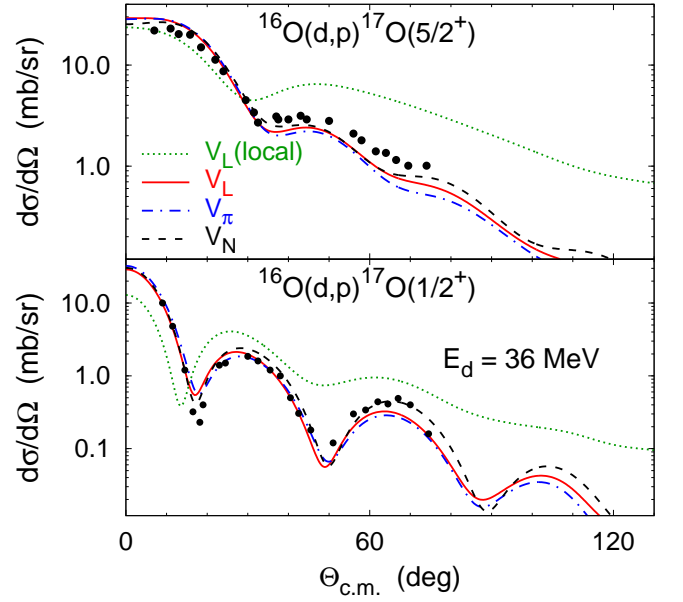


FIG. 5. (Color online) Differential cross section for ${}^{16}\text{O}(d,p){}^{17}\text{O}$ transfer reactions at $E_d = 36$ MeV leading to ${}^{17}\text{O}$ ground $\frac{5}{2}^+$ (top) and excited $\frac{1}{2}^+$ (bottom) states. Predictions obtained using local L -dependent OP from Ref. [10] (dotted curves), nonlocal L -dependent OP (solid curves), nonlocal π -dependent OP (dashed-dotted curves), and nonlocal L -independent OP (dashed curves) are compared with experimental data from Ref. [30].

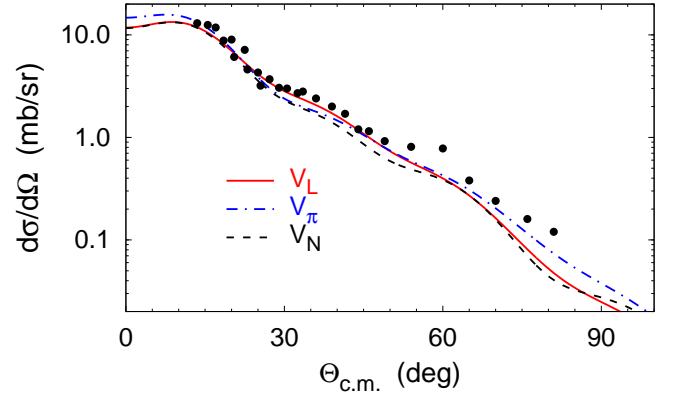


FIG. 6. (Color online) Differential cross section for ${}^{16}\text{O}(d,p){}^{17}\text{O}$ transfer to ${}^{17}\text{O}$ ground state $\frac{5}{2}^+$ at $E_d = 63.2$ MeV. Predictions obtained using nonlocal L -dependent OP (solid curves), nonlocal π -dependent OP (dashed-dotted curves), and nonlocal L -independent OP (dashed curves) are compared with experimental data from Ref. [30].

be quite similar to L - and π -dependent models of Sec. II and Ref. [10]. This may indicate that deuteron tensor analyzing powers are not well constrained by nucleon-nucleus A_y data.

Next we study ${}^{16}\text{O}(d,p){}^{17}\text{O}$ transfer reactions. In this

case the potential must support bound state for the final nucleus ^{17}O . We therefore take real binding potentials from Ref. [31] in $n + ^{16}\text{O}$ partial wave $\frac{5}{2}^+ (\frac{1}{2}^+)$ when calculating transfer to ^{17}O ground state (excited state). Differential cross section results for both reactions at $E_d = 36$ MeV are shown in Fig. 5. By comparing with the experimental data [30] one notices the failure of the local L -dependent OP from Ref. [10] over the whole angular regime. In contrast, all nonlocal OPs, L - or π -dependent or not, provide quite good description of the experimental data. Thus, L - and π -dependent terms in the OP appear to be quite irrelevant while the nonlocality of the OP turns out to be essential. Similar findings regarding the OP nonlocality in transfer reactions emerged in Refs. [14, 31] where a broader range of reactions was investigated. Note that there is a difference between present calculations and those of in Ref. [14] in the choice of $n + ^{16}\text{O}$ potential: it was real in Ref. [14] but complex here (except for ^{17}O bound state partial wave). In Fig. 6 we present one more example, i.e., $^{16}\text{O}(d, p)^{17}\text{O}$ transfer to ^{17}O ground state $\frac{5}{2}^+$ at $E_d = 63.2$ MeV, not considered in Refs. [14, 31]. Again, the account of the experimental data [30] by all employed nonlocal potentials is quite good, while the calculations of Ref. [18] with local but explicitly energy-dependent potentials heavily failed in reproducing this observable.

V. SUMMARY AND CONCLUSIONS

We developed a number of angular-momentum or parity-dependent optical potentials for nucleon- ^{16}O system. Those nonstandard additional terms enabled to fit elastic nucleon-nucleus scattering data at large angles. However, the parameters turn out to be energy-dependent. Optical potential with energy-independent parameters is able to fit two-body data very well around the chosen energy in the whole angular regime for the differential cross section and up to about $\Theta_{\text{c.m.}} = 100^\circ$ for the analyzing power. At more distant energies the de-

scription remains good at not too large scattering angles $\Theta_{\text{c.m.}} \leq 100^\circ$. The local L -dependent OP from Ref. [10] turns out to be much stronger energy-dependent, with a fixed parameter set able to account for the data in narrow energy region only. In this respect the potentials of the present work represent a significant improvement.

The explicit angular-momentum or parity dependence of the OP can be handled in the Faddeev/AGS three-body scattering equations solved in the momentum-space partial-wave representation where each two-body potential and the corresponding transition matrix is calculated in its proper basis. In an energy-independent form the nonlocal L - or π -dependent potentials were used to calculate differential cross section and deuteron analyzing powers for $d + ^{16}\text{O}$ elastic scattering and $^{16}\text{O}(d, p)^{17}\text{O}$ transfer reactions. To isolate the effect of L - or π -dependence and nonlocality, same observables were calculated using local L -dependent [10] and nonlocal L -independent [17] potentials. In all considered reactions nonlocal OPs provide quite similar and reasonable description of differential cross section data. In contrast, the predictions using the local L -dependent OP [10] strongly deviate from the data and all nonlocal OPs for $\Theta_{\text{c.m.}} \geq 50^\circ$ in $d + ^{16}\text{O}$ elastic scattering and in the whole angular regime for $^{16}\text{O}(d, p)^{17}\text{O}$ transfer reactions. Based on this fact we conclude that L - and π -dependent terms in the OP may be quite irrelevant for three-body scattering but the nonlocality plays a major role, especially in transfer reactions; the latter finding is in accordance with Refs. [14, 31–33]. The comparison of predictions and data for deuteron analyzing powers in $d + ^{16}\text{O}$ elastic scattering is less conclusive. The agreement is reasonable for all properly fitted models at not too large scattering angles $\Theta_{\text{c.m.}} \leq 60^\circ$, but beyond the predictions may deviate from data and from each other. Furthermore, a proper fit to two-body analyzing power data appears to be relevant for deuteron vector analyzing power A_y , but not for tensor analyzing powers A_{yy} , A_{xx} , and A_{xz} .

This work was supported by Lietuvos Mokslo Taryba (Research Council of Lithuania) under Contract No. MIP-094/2015.

-
- [1] R. C. Johnson and P. J. R. Soper, Phys. Rev. C **1**, 976 (1970).
 - [2] N. Austern, Y. Iseri, M. Kamimura, M. Kawai, G. Rawitscher, and M. Yahiro, Phys. Rep. **154**, 125 (1987).
 - [3] A. Deltuva, A. M. Moro, E. Cravo, F. M. Nunes, and A. C. Fonseca, Phys. Rev. C **76**, 064602 (2007).
 - [4] B. A. Watson, P. P. Singh, and R. E. Segel, Phys. Rev. **182**, 978 (1969).
 - [5] J. J. H. Menet, E. E. Gross, J. J. Malanify, and A. Zucker, Phys. Rev. C **4** 1114, (1971).
 - [6] F. D. Becchetti Jr. and G. W. Greenlees, Phys. Rev. **182**, 1190 (1969).
 - [7] R. L. Varner, W. J. Thompson, T. L. McAbee, E. J. Ludwig, and T. B. Clegg, Phys. Rep. **201**, 57 (1991).
 - [8] A. J. Koning and J. P. Delaroche, Nucl. Phys. **A713**, 231 (2003).
 - [9] A. Deltuva, Phys. Rev. C **88**, 011601(R) (2013).
 - [10] A. M. Kobos and R. S. Mackintosh, J. Phys. G **5**, 97 (1979).
 - [11] R. S. Mackintosh and A. M. Kobos, J. Phys. G **5**, 359 (1979).
 - [12] G. H. Rawitscher and D. Lukaszek, Phys. Rev. C **69**, 044608 (2004).
 - [13] S. G. Cooper, Nucl. Phys. A **618**, 87 (1997).
 - [14] A. Deltuva, Phys. Rev. C **79**, 021602(R) (2009).
 - [15] J. M. Cameron, J. R. Richardson, W. T. H. van Oers, and J. W. Verba, Phys. Rev. **167**, 908 (1968).
 - [16] H. B. Eldridge, S. N. Bunker, J. M. Cameron, J. R.

- Richardson, and W. T. H. van Oers, Phys. Rev. **167**, 915 (1968).
- [17] M. M. Giannini and G. Ricco, Ann. Phys. (NY) **102**, 458 (1976).
- [18] A. Deltuva and A. C. Fonseca, Phys. Rev. C **79**, 014606 (2009).
- [19] Parameter tables and computer codes can be obtained from authors upon request.
- [20] P. Grabmayr, J. Rapaport, and R. Finlay, Nucl. Phys. A **350**, 167 (1980).
- [21] S. T. Lam *et al.*, Phys. Rev. C **32**, 76 (1985).
- [22] J. P. Delaroche, M. S. Islam, and R. W. Finlay, Phys. Rev. C **33**, 1826 (1986).
- [23] S. G. Cooper and R. S. Mackintosh, Nucl. Phys. A **592**, 338 (1995).
- [24] E. O. Alt, P. Grassberger, and W. Sandhas, Nucl. Phys. **B2**, 167 (1967).
- [25] J. R. Taylor, Nuovo Cimento B **23**, 313 (1974); M. D. Semon and J. R. Taylor, Nuovo Cimento A **26**, 48 (1975).
- [26] E. O. Alt and W. Sandhas, Phys. Rev. C **21**, 1733 (1980).
- [27] A. Deltuva, A. C. Fonseca, and P. U. Sauer, Phys. Rev. C **71**, 054005 (2005).
- [28] R. Machleidt, Phys. Rev. C **63**, 024001 (2001).
- [29] N. Matsuoka *et al.*, Nucl. Phys. **A455**, 413 (1986).
- [30] M. D. Cooper, W. F. Hornyak, and P. G. Roos, Nucl. Phys. **A218**, 249 (1974).
- [31] A. Deltuva, Phys. Rev. C **92**, 064613 (2015).
- [32] N. K. Timofeyuk and R. C. Johnson, Phys. Rev. C **87**, 064610 (2013).
- [33] L. J. Titus, F. M. Nunes, and G. Potel, Phys. Rev. C **93**, 014604 (2016).